Mentor Support Booklet

Giving subject specific feedback & setting targets

in

MATHEMATICS

CONTENTS

Giving subject specific feedback & setting targets in mathematics

At York St John University we aim to work in partnership with our mentors in supporting student teachers to develop confidence and competence in the primary mathematics classroom – high quality, subject specific feedback and the setting of achievable and helpful targets whilst on school placement are central to this support.

The aims of this booklet include:

- **Making links between key centre-based messages and the subject specific feedback student teachers receive following appraised and on-going maths sessions** – *we feel that it is vital for mentors to be fully aware of the key messages shared with student teachers so that they can praise student teachers who are taking risks in these areas and can provide advice on how to put the theory into practice*
- **Support mentors, who may not be maths specialists, in providing high quality subject specific feedback following appraised maths lessons or at *any* point throughout a school experience** – *high quality, subject specific, feedback often requires the mentor to look at the specific area of mathematics being addressed, rather than keeping comments generic*
- **Support mentors in the setting of achievable and helpful maths specific targets –** *we are keen for targets to show not only 'what' to develop but also 'how' to improve, taking into account the age/level of the children.*
- **Support student teachers in making links between pedagogical theory and classroom practice**

This booklet contains a series of prompt sheets (double sided) which focus on specific areas of the primary mathematics curriculum.

- Understanding Number: Counting, Comparing and Ordering
- Number: Calculating (Addition & Subtraction)
- Number: Calculating (Multiplication & Division)
- Algebraic Thinking
- Shape
- Measures
- Data Handling

It also contains an opening section on 'Generic mathematical pedagogy (including U&A)'. No prompt sheet has been included for this section as it was felt that the prompt sheet already available for maths covers much of what is needed.

Each prompt sheet has been produced to include the following:

- **The key YSJ messages supported by research findings**

A series of quotes from academically acclaimed research is provided in each section. These highlight key aspects of effective pedagogy in the specific area of the maths curriculum being considered. These are further underpinned by the key messages shared with student teachers during centre-based inputs at YSJ.

➢ Mentors can use these key messages to guide their subject specific feedback in discussion with the student teachers, considering key strengths and areas that could strengthen or broaden the student teachers practice in the specific area of maths being taught, perhaps encouraging student teachers to further their

own understanding through relevant reading.

- Student teachers can use the key messages and reference material when planning to help them re-consider what constitutes effective pedagogy in the specific areas being taught.

- **Prompts from the DCSF publications**

These show how the YSJ key messages relate to specific DCSF publications. The prompts provide a starting point for setting targets that are not only subject/topic specific but also age appropriate (taking into account the level at which the mathematics is being explored).

- Mentors can use these prompts to set achievable and helpful targets that will broaden the student teachers' practice in the chosen area of mathematics. If the observed lesson is part of a series of lessons on the topic this will provide the student teacher with an opportunity to address targets immediately.

- Student teachers can use these prompts to consider the level of work they are planning, and to support their explanations and reasoning during the discussion with the mentor following an appraisal – helping them to articulate their decisions.

The last two pages of the booklet are copies of a document circulated to student teachers in an effort to help them understand how to formulate clear success criteria when planning maths lessons. This is something that many student teachers find difficult, often just re-iterating learning objectives, and we thought that mentors may find it helpful to know exactly what messages we are giving about this aspect of planning.

It should also be noted that due to timing student teachers may not have covered specific areas of mathematics at the university prior to a school placement and <u>may, therefore, need additional support with any planning for lessons based on these areas.</u> The following table provides an approximate guide to coverage by the different programmes at different stages.

Aspects covered by the start of School Experience

	1	2	3
Undergraduate Programme	• Early number • Place value • Calculation strategies • Shape • Measurement • Session planning	As for SE1 plus: • Fractions, decimals, %, proportion and ratio • Algebraic thinking (sequences; equations; generalising) • Graphing equations • Data Handling • Probability • Weekly planning for a single and mixed age class • Unit planning • Inclusion (SEN, G&T, EAL, gender) • Assessment (AFL, AOL)	As for SE1 and SE2
Postgraduate Programme (Full Time)	• Discussion based maths • Maths games • Early number • Place value • Calculating (Pt 1) • Problem solving • Session planning	• Calculating (Pt 2) • FDRPP and algebra • Data handling • Inclusion • Review of session planning • Weekly planning	• Number revisited • Shape and space • Measures • Maths outside the classroom • Planning and assessing
Postgraduate Programme (Part Time)	• Early number • Discussion based maths • Maths games • Place value	• Calculations • FDRPP • Planning and assessing	• Problem solving • Data handling • Measures • EAL • Shape • Planning and assessing • Maths trails

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- Ollerton, M. (2008a) **A Philosophy For Learning** (paper written for the YSJ 'Inspiring Mathematics Champions' project)
- Ollerton, M. (2008b) **A pedagogy for using problem solving approaches to teach and learn mathematics in primary schools** *(Appendix to the handbook for the YSJ 'Inspiring Mathematics Champions' project)*
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Generic mathematical pedagogy

The key York St John messages and associated research findings

'...if children are brought up on a diet of problem-solving approaches in their learning, where opportunities naturally exist for them to be resilient, to persevere and to try to overcome difficulties they will simultaneously develop independence and a capability to share with and support others.' (Ollerton, 2008a, p.1)

'...it is through *using* mathematics in the context of other subjects that pupils develop their ability to apply mathematics. In these contexts they ...learn to appreciate the role that maths plays in the real world and how it contributes to their understanding of other subjects.' (Coles & Copeland, 2002, p.7)

'We want mathematical discourse in our classrooms to be high quality. Creating tasks for children to talk about, and demanding that they take care over spoken language can help us to do this.' (Pratt, 2006, p. 30)

'Some children find it a lot easier to explain their thinking if they have equipment available to touch and move so that they can describe their actions rather than try to formulate abstract thoughts. (Anghileri, 200, p.90)

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a) **The teaching of mathematics should include a balance of the following strategies:**

- **The integration of AT one (using and applying) when teaching all other strands at KS1 and 2.**
- **Clear expositions and detailed modelling of tasks and calculation strategies.**
- **The use of a range of appropriate mathematical vocabulary by both the teacher and the children.**
- **Open ended questioning to extend understanding of concepts.**
- **Chances for all the class to contribute answers.**
- **Use of practical apparatus to support children's calculations.**
- **Appropriate practical and kinaesthetic work.**
- **An investigative approach *(both this and the above point emphasising the importance of using and applying mathematical knowledge and skills)* – i.e. allowing children chances to discover rules for themselves by looking for patterns.**
- **Problem solving activities of various types *e.g. word, logic, patterns & rules, finding all possibilities*.**
- **Discussion based learning.**
- **Collaborative work.**
- **The use of games and puzzles.**
- **A creative approach - link the mathematical content to other work being done in other areas of the curriculum; exploit opportunities for mathematics in settings other than the classroom.**

b) **Remember to give the children thinking time and ensure all children are counting together rather than having a 'spotlight' put on them.**

c) **In a FS setting – exploit all provision areas for mathematical opportunities.**

Understanding Number: Counting, Comparing and Ordering

The key York St John messages and associated research findings

'Since children find counting an intrinsically satisfying activity, there need to be many opportunities for counting to be practised in the different contexts... [teachers should focus] on knowing which [counting] procedure a child is habitually using to make some judgement as to whether [it] is enabling or hindering the child' (Maclellen, 1997, p.40)

'Counting is one of the first mathematical concepts that children learn. Children learn to count both formally and informally through interaction with others and their environment' (Lawton, 2005, p23)

'Children will encounter the decimal point as a separator in the context of money… and then in the context of length ….They can use the notation…initially without having any real awareness of figures representing tenths and hundredths.' (Haylock, 2010, p.77)

'In the explaining of place value to children use the language of 'exchanging one of these for ten of those' as you move right to left along the powers of ten, and 'exchanging ten of these for one of those' as you move left to right.' (Haylock, 2010, p.72)

'We know that:

- 15 million adults in the UK have very poor numeracy skills
- One in six companies currently have to provide remedial mathematics classes
- Numeracy failure starts early – each year around 35,000 eleven year olds (6% of their age group) leave primary school with numeracy skills at or below the level expected of the average seven year old
- Numeracy failure carries high social costs – the proportion of the prison population with very poor numeracy skills, for example, is even greater than the proportion with poor literacy skills' (Every Child Counts, Internet)

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- **Counting forms a regular part of children's daily oral and mental work.**
- **Use a variety of counting activities rather than just counting in multiples from 0 to 10x and back *(e.g. if the middle of the stick is 30 what might the start and end numbers be? Why?)***

- **Use a counting stick to support the teaching of counting – hold it in the centre; ensure you count in the right direction for the children; don't always have the start of the stick as 0; remember that negative numbers can be included.**
- **Consider the use of a counting hoop to allow counting to continue on and on.**
- **Highlight the link with place value when ordering and comparing numbers.**
- **Help children appreciate links between fractions, decimals, percentages etc. *(i.e. they are all different ways of expressing the same amount)***
- **Ask children to look at the most significant figure when ordering numbers – this can be compared to the use of alphabetical order.**
- **Introduce examples to challenge misconceptions – not just the ones that "fit the rule" nicely *(e.g. the misconception that 'the longer the decimal fraction the bigger the number' may be masked if you ask the children 'which is smaller: 1.247 or 1.1? They may get the right answer for the wrong reason).***

Here are some prompts from the DCSF publications ' Children thinking mathematically: PSRN essential knowledge for Early Years Practice' and 'Securing Levels 2 & 4 in Mathematics' to support in providing subject specific feedback and targets for the teaching of counting, comparing and ordering:

'Children thinking mathematically: PSRN essential knowledge'	**'Securing level 2 in mathematics'**	**'Securing level 4 in mathematics'**
Make sure that: • Practitioners plan to introduce rhymes systematically, so that children can increase their repertoire of favourite rhymes • Rhymes include: counting back and counting forwards; 'no' or 'none' (five little ducks); counting in pairs (2,4,6,8, Mary at the cottage gate); counting in five, ten and beyond • Children have opportunities to revisit rhymes independently – through small –world play, with puppets, soft toys, and other props • As children counting skill develop you provide opportunities for children to begin to understand one-to-one correspondence, stable order, abstraction, cardinality and order irrelevance • Once children begin to count fluently that they can recognise up to three objects without counting Subitising is the term used when children can judge the number of objects in a group rapidly, accurately and confidently without counting them. Many children subitise when looking at dice and patterns • Note when children self-check their count as they will have moved on from the preliminary counting skills. Some children are helped at this stage by the practitioner modelling this process in an appropriate context • Display number lines because children need to have a reference for their conversations. Number tracks are also useful because children can jump on them or put object on them	Make sure that: • counting forms a regular part of children's daily oral and mental work • you use models and images to demonstrate place value and the relative value of numbers • children use equipment such as bead strings, partially marked number lines, place-value cards and base-ten apparatus to secure their understanding of place value • you build in frequent opportunities for children to explain their reasoning, for example, describing patterns in a sequence and how they would use these to predict the next term • children solve practical problems involving estimating, counting, ordering and rounding numbers • children position two-digit numbers onto partially marked number lines to support their decisions when ordering numbers or rounding them to the nearest 10 • children see, manipulate and generate examples of halves and quarters, using practical equipment to represent parts of a whole	Make sure that: • you regularly include decimal number facts and counting in daily oral and mental work • you include counting up in time intervals in your mental and oral work • children regularly use partially-numbered and blank number lines as part of daily oral and mental work • you use models such as place value charts or grids to teach children about place value • you watch for children who believe that the column to the left of thousands represents millions • you stress that the value of each column gets ten times bigger as you move to the left • children describe the value of decimal digits using the language of both decimals and fractions, *for example, 'nought point nought two' and 'two hundredths'* • you reinforce decimal place value using visual models such as bead strings or base 10 apparatus and using contexts such as length and money • you encourage them to use accurate place value language to describe each step • children do not describe the effect of multiplying by 10 as 'adding a nought' • children appreciate that the units of time are not decimal, so they need to take care with time calculations • children link money notation to decimal place value, understanding, *for example, that the 2 in £4.27 has value of 2 tenths of a pound or 20p*

Number: Calculating (Addition & Subtraction)

The key York St John messages and associated research findings

'...[A]ll calculations for children in the first few years of schooling should, ideally, be done mentally, using whatever aids they feel they need.' (Thompson, 1997, p.98)

'...[L]earning key facts "by heart" enables children to concentrate on the calculation which helps them to develop calculation strategies. Using and applying strategies to work out answers helps [them] to acquire and so remember more facts.' (DfE, 2010, p.13)

'[T]he use of standard mathematical symbols is something that develops slowly in young children. They should be exposed to the symbols, but not obliged to use them, until they feel comfortable with them.' (Thompson, 1997, p.98)

'Where tasks were presented horizontally (e.g. 24+43) children visualised rearrangements of individual numbers, but when tasks were presented vertically, they fell back on the use of algorithmic methods...[exhibiting] rule-like behaviour which they clearly did not always understand.' (Nickson, 2000, p.21)

'Once children can use counting methods efficiently to solve simple arithmetic problems, the next potential stumbling block is place value....Children need a sound understanding of [this] if they are to move to ...more abstract computational methods.' (Lawton, 2005, p.54)

- **Use a variety of models and images to help children understand the concepts of addition and subtraction.**
- **Model different calculation strategies carefully – use number lines and number squares.**
- **Encourage discussion of different calculation strategies – there is no one 'right way' to find an answer – some ways are more efficient but many are valid.**
- **Use open rather than closed examples. (*e.g. 'Which two numbers can you add to get 1?' rather than 'What is 3 +4?'*).**
- **Relate to real life – provide reasons for calculating.**
- **Introduce examples to challenge misconceptions – not just the ones that "fit the rule" nicely *(e.g. 34 +47 rather than 34 + 41).***
- **Make use of practical resources to support calculations - provide a range of models. (e.g. counters and a number line to model counting on to find the difference).**
- **Show the concept rather than just algorithms. *(e.g. do they understand that subtraction can be viewed as counting on to find the difference; do they know that addition is the bringing together of 2 or more sets to find how many altogether?).***
- **Highlight the link with place value in all calculation strategies used.**
- **Always present problems horizontally (e.g. 38 – 22 = ?)**

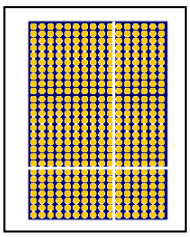
Here are some prompts from the DCSF publications ' Children thinking mathematically: PSRN essential knowledge for Early Years Practice' and 'Securing Levels 2 & 4 in Mathematics' to support in providing subject specific feedback and targets for addition and subtraction.		
'Children thinking mathematically: PSRN essential knowledge' Make sure that: • Knowing mathematical vocabulary is important for children to have the language tools for mathematics. • Maths vocabulary is set in purposeful contexts and not taught as a list to be remembered • Open questions are used to encourage and support children's problem solving reasoning and creative thinking in mathematics • Look for problems that naturally occur in children's play to provide contexts in which children can use their developing mathematical ideas and methods – if the child is told the method to use, then they are not problem solving • Children's shared talk about problem solving does not always lead to mathematical graphics; the use of language is valuable in its own right • Children compare groups or quantities using one-to-one correspondence to find the difference – early subtraction • Children count back from a fixed number when taking away – subtraction • Children count on from a fixed number when combining two groups of objects – addition • Children find one more or two fewer by counting on or back – early addition and subtraction	***Securing level 2 in mathematics'*** Make sure that: • daily oral and mental work involves generating subtraction facts from addition facts and vice versa • children have regular opportunities to explain how they decided whether to use addition or subtraction to solve particular problems • you model the correct use of addition and subtraction vocabulary in a variety of contexts and support children towards using the language to explain their thinking and methods • you model how to represent practical situations, using apparatus, pictorial representation and then using number sentences • children have opportunities to choose when to work out subtraction, using a counting back or a counting on method • children experience situations and problems where they need to use inverse operations, for example, to *undo* an operation or to find a missing number, and see these operations modelled • children rehearse and use addition and subtraction facts regularly in daily oral and mental work • children use partially marked number lines and then begin to draw their own number lines to record addition and subtraction methods • you plan frequent opportunities for children to explain their methods and to discuss and compare alternative methods • you model different ways of recording mental methods and encourage children to use pictures and jottings to help them carry out mental calculations • children regularly receive opportunities to use addition and subtraction to solve practical and word problems, including those involving money and measures • you discuss with children how the numbers involved in a calculation might lead to a particular method and why it is appropriate	***'Securing level 4 in mathematics'*** Make sure that: • children have regular opportunities to describe their methods of solving calculations and problems involving decimals, time, money to each other and you • children consider whether calculations can be done mentally before deciding on a written method. *(Counting on, for example, is often an effective method for working out change)* • you exploit opportunities to solve real problems, for example getting the children to calculate the pricing and schedule for a school visit • children appreciate that the units of time are not decimal, so they need to take care working out time intervals *(time lines are the best model for this)*

Number: Calculating (Multiplication and Division)

The key York St John messages and associated research findings

'When primary school children are to be taught a method for multiplying two-digit numbers, encourage the method based on the areas of four rectangles, splitting each of the numbers into tens and units. Once they are fluent with this, they will not need to draw the rectangle and can record the steps in a grid.' (Haylock, 2010, p.154)

e.g. 26 x 16 = (20x10) + (20x6) + (6x10) + (6x6)



'Experiences with number lines will also help children to understand the nature of multiplication and division as counting groups is related to a visual image of *equal jumps*.' (Anghileri, 1997, p.49)

'Children's first experience of multiplication arises where they make groups with equal numbers of objects and recognize the possibility of counting the groups rather than counting individual items. Where there is a natural link between the objects, like pairs of shoes or sets of wheels for model cars, this counting of groups is easiest.' (Anghileri, 1997, p.41)

'The arithmetic operation of division is introduced in school initially as a formalisation of sharing and this model holds an enduring position in pupils' intuitive thinking (Fischbein et al., 1985). Although this serves as one possible model, an equally powerful interpretation relates division to finding the number of equal groups in a given total.' (Anghileri, 2001, p.86)

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- **Make use of practical resources to support calculations and use a variety of models and images to help children understand the concepts of multiplication and division. *(e.g. multiplication can be viewed as a row of people wearing pairs of shoes / gloves; sets of the same number of objects; an array of counters; a grid of squares)***
- **Model different calculation strategies carefully – use number lines, number squares and grids.**
- **Encourage discussion of different calculation strategies – there is no one 'right way' to find an answer – some ways are more efficient but many are valid.**
- **Use open rather than closed examples (*e.g. 'Which two numbers can you multiply to get 12?' rather than 'What is 3 x 4?'*).**
- **Relate to real life – provide reasons for calculating.**
- **Introduce examples to challenge misconceptions – not just the ones that "fit the rule" nicely *(e.g. 20% of 500 rather than 10% of 500 – this would ensure that the 'divide by the %' over-generalisation from finding 10% is not masked).***
- **Show the concept rather than just algorithms *(e.g. do they understand that division can be viewed as repeated subtraction, rather than just how to do it? Model as chunking on a number line.)***
- **Highlight the link with place value in all calculation strategies used.**

Here are some prompts from the DCSF publications ' Children thinking mathematically: PSRN essential knowledge for Early Years Practice' and 'Securing Levels 3 & 4 in Mathematics' to support in providing subject specific feedback and targets for the teaching of multiplication and division:		
'Children thinking mathematically: PSRN essential knowledge' Make sure that: • Knowing mathematical vocabulary is important for children to have the language tools for mathematics • Maths vocabulary is set in purposeful contexts and not taught as a list to be remembered • Open questions are used to encourage and support children's problem solving reasoning and creative thinking in mathematics • Look for problems that naturally occur in children's play to provide a context to provide contexts in which children can use their developing mathematical ideas and methods – if the child is told the method to use, then they are not problem solving • Children's shared talk about problem solving does not always lead to mathematical graphics; the use of language is valuable in its own right • Sharing objects equally by counting how many in each group – early division • Count groups of the same number of objects and add them together – early multiplication	**Securing <u>level 3</u> in mathematics' (No section for x and ÷ in level 2 booklet)** Make sure that: • children rehearse multiplication and division facts regularly through daily oral and mental work • children use multiplication rather than continuing to use inefficient repeated addition • children can use known facts to multiply or divide multiples of 10*, working out, for example, 30 x 5 or 280 ÷ 4* • children are able to recognise when a word problem involves multiplication or division • children recognise 'special case' calculations*, for example, dividing by 4 by halving and halving again* • children can find missing numbers in calculations *such as □ ÷ 5 = 12 and 180 ÷ □ = 30* • children have regular opportunities to explain and compare calculation methods • children associate the language of division with finding a fraction, *for example, they associate dividing by 5 with finding a fifth* • you use children's understanding of mental methods as the basis for the development of written methods such as the grid method and chunking	***'Securing level 4 in mathematics'*** Make sure that: • children have regular opportunities to describe their methods of solving calculations and problems involving decimals, time, money to each other and you • children consider whether calculations can be done mentally before deciding on a written method. *(e.g. can they use their table or doubling facts and multiplying by 10 or 100 to solve 23 x 20?)* • you exploit opportunities to solve real problems, for example getting the children to calculate the pricing and schedule for a school visit • children use real data. Collect price lists and timetables such as from the local sports centre children are familiar with common shopping terminology including: *Best buy , 3 for the price of 2* and *price per 100g* • children understand division as grouping, for example reading 100 ÷ 7 as *'How many 7s can be made from 100?'* • you pick up on common calculation errors. Ask children to identify what has gone wrong and suggest strategies to use, *e.g. discuss why 1.2 ÷ 6 is not 2*

Algebraic Thinking

The key York St John messages and associated research findings

'Using a problem solving pedagogy...requires the teacher to use less didactic methods and employ a greater range of facilitative approaches. As such teaching must, in some ways, be subordinated to learning. Broadly speaking the role of the mathematics teacher is to help learners develop skills inherent within problem solving: organising, analysing and generalising...' (Ollerton, 2008

'...a great deal of algebra is to be found in the objectives identified within the 'using and applying' strand... Algebra is firmly rooted in patterns, and the creation, extension and expression of patterns is fundamental to algebra.' (Mooney et al 2007, p.137)

'Alongside the teaching of number sequences, children also start to investigate patterns within shape and space. They start by recreating patterns, then extending them; they then talk about the patterns and go on to describe them. This ...leads to a realisation that number patterns can be generated by spatial patterns...[which] can be expressed generally, at first in words and then using symbols.' (Mooney et al 2007, p.139)

'The meaninglessness of the problems that pupils are generally faced with are a cause for concern...[they] do not invite pupils' involvement essentially because they do not arise from any of the pupils' activity and they cannot relate them to anything...tangible in their lives.' (Nickson, 2000, p. 123)

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- **The focus should be on getting the children to think algebraically and look for patterns in sequences *e.g. deciding what comes next? Why? Finding unknowns and use what has been given.***
- **Ensure children understand that an equals sign signifies a balancing of both sides of a number sentence. *(i.e. the left hand side is not 'the answer'*) Use a practical model of this *(e.g. an equaliser bar or a two pan balance).***
- **Encourage children to view number sentences as a record of what is happening in a function machine and to draw pictures to show what a function machine is doing.**
- **Encourage the use of a picture/physical model to find unknowns (*e.g. to solve 3 + ? = 10 match 3 counters underneath a row of 10 then count on to see how many more are needed).***
- **Model working systematically *(e.g. starting with the first model in a set then the next biggest and so on).***
- **Help children appreciate links between fractions, decimals, percentages etc. *(i.e. they are all different ways of expressing the same amount).***
- **Plan for hands-on activities, avoiding the purely abstract *(e.g. gather data for sequences by counting items within ever growing sets).***
- **Encourage older children to look at diagrams that they have drawn to find a proof for their rules (i.e. why does the rule work?) Just trying out more examples that are successful does not establish a proof.**

Here are some prompts from the DCSF publications 'Children thinking mathematically: PSRN essential knowledge for Early Years Practice', ''Overcoming Barriers to Mathematics – L1-2', 'Securing Level 2 in Mathematics', 'Securing Level 5 in Mathematics' and 'Overcoming Barriers to Mathematics – L4-5' to support in providing subject specific feedback and targets for algebraic thinking.

'Children thinking mathematically: PSRN essential knowledge'	***'Securing level 2 in mathematics'***	***'Overcoming barriers in mathematics – L4-5'***
Make sure that: • you provide opportunities for problem solving, reasoning, critical thinking and reflection ***'Overcoming Barriers to Mathematics – L1-2'*** Make sure that you model how to: • solve problems by 'thinking aloud', describing the clues that helped you to identify an appropriate strategy to solve the problem and what you might record • represent practical situations, using apparatus, pictures and number sentences, and encourage children to choose how to represent problems	Make sure that: • you build in frequent opportunities for children to explain their reasoning, for example, describing patterns in a sequence and how they would use these to predict the next term • children experience situations and problems where they need to use inverse operations, for example, to *undo* an operation or to find a missing number, and see these operations modelled	Encourage children to: • make generalisations about sequences and explain why given numbers do or do not belong to the given sequence • create an algebraic expression that describes a simple relationship ***'Securing level 5 in mathematics'*** Make sure that: • children compare solutions to problems, discussing different representations to see which is most effective and sharing alternative explanations and reasoning • children discuss 'always/sometimes/never' statements, such as *all multiples of 6 are also multiples of 3* or *all prime numbers are odd*, and justify their decisions • you discuss how to use numbers, diagrams and symbols, in addition to words, when writing a reasoned explanation to make recording more concise • you model the language of reasoning, for example, suggesting how to word a general statement • you teach children how to use counter-examples to disprove a hypothesis such as *all factors of 20 are even*

Shape and space

The key York St John messages and associated research findings

'Van Hiele (1986) offers a widely accepted framework to explain how pupils develop in geometric understanding [Level 1– Visualisation; Level 2– Analysis; Level 3- Informal Deduction] ... Children tend to move between [these] levels flexibly, depending on the task and shapes being studied, rather than ...hierarchically.' (Hansen, 2005, p.78)

'...through building, stacking, rolling, sliding and handling shapes of all kinds [young] children will begin to acquire a sense of what exactly constitutes any given shape.' (Marsh & Baker, In Basford & Hodson, 2008, p61)

'...after working in LOGO contexts designed to address ideas of angle and turn, children develop mathematically correct, coherent, and abstract ideas about these concepts.' (Clements and Battista, 1990, p314)

'... children can be helped to develop the skills of thinking about a problem by forming and manipulating an image in their heads or visualising.' (Hopkins et al., 1999, p 72)

'It is our responsibility as teachers to ensure that children are exposed to shapes of varying size, orientation and type, to broaden their limited understanding of shape.' (Hansen, 2005, p77)

'Give children opportunity to explore the properties of various shape, including the different kinds of triangles and quadrilaterals, and regular and irregular shapes, by folding, tracing, matching, looking for reflective and rotational symmetries, and drawing out the implications of these.'(Haylock, 2010, p 317)

- **Ensure very young children have a range of opportunities to explore the shape of things around them.**
- **Ensure that visual aids present shapes in different orientations and use non-prototypical shapes.**
- **Make use of a good range of resources *(e.g. it is difficult for children to draw shapes so use geoboards, dotted paper and geostrips instead; ensure mirrors are available to check for symmetry)***
- **Avoid the use of the IWB to provide representations of 3D shapes – allow children to handle real examples.**
- **Look at shapes in context rather than as abstract, plastic shapes.**
- **When asking children to identify shapes ensure that they first look carefully at all of the properties involved. Avoid them thinking of shapes within just single categories. *(e.g. a square is also a special type of rectangle, rhombus, parallelogram etc.)***
- **Exploit speaking and listening opportunities to make use of a wide range of appropriate vocabulary. *(e.g. use visualisation exercises, look at partially drawn shapes – what might they be part of? Why? What couldn't they be? Why not?)***
- **Use LOGO packages to provide problem solving situations in shape and direction – ensure children are prepared for working on screen with earlier 'pre-logo' activities (e.g. giving instructions to move along a route on a map)**

Here are some prompts from the DCSF publications ' Children thinking mathematically: PSRN essential knowledge for Early Years Practice' and 'Securing Levels 2 & 4 in Mathematics' to support in providing subject specific feedback and targets for the teaching of shape.		
'Children thinking mathematically: PSRN essential knowledge for Early Years Practice' **Make sure that:** • children are given first hand explorations to allow them to use their senses: playing with natural and everyday resources. • settings provide a wide range of opportunities both inside and outside for children to explore space on different levels and in different ways • a child's use of everyday language should be accepted and valued… It is important that children use language that makes sense to them • the naming of shapes can be less important than… hands on exploration and should not be introduced in isolation as a discrete activity.	***'Securing level 2 in mathematics'*** **Make sure that:** • shape and space activities are included in oral and mental work and include opportunities for discussion and reasoning • children have opportunities to create 2-D shapes, including use of squared and isometric paper • children have opportunities to create 3-D models, using construction kits, and to explore and discuss their properties • children experience practical hands-on activities involving shapes rather than simply responding to representations of them • you build on children's experience of creating, drawing and manipulating shapes, helping them to visualise familiar shapes by hiding and revealing the shapes • you model the use of mathematical language, display shape vocabulary and encourage children to use it accurately • children are given opportunities to recognise and talk about shapes in a variety of contexts, including photographs, pictures, puzzles and the indoor and outdoor environment • children discuss the properties of the same shape when it is placed in different orientations or enlarged	***'Securing level 4 in mathematics'*** **Make sure that:** • children regularly solve shape problems • you include shape and space activities in your daily oral and mental work, especially visualisation • you display shape vocabulary and model how it is used. Plan talk-rich activities where children have to use the vocabulary accurately • children recognise shapes in different orientations • children have lots of opportunities to handle physical 2-D and 3-D shapes • children have opportunities to reflect shapes in diagonal mirror lines. Ask children to explain how they know the exact point where each vertex will be reflected to • children recognise irregular shapes such as irregular hexagons • children know how to use mirrors to check reflection and how to use tracing paper to check rotation • children use different grids (square, triangular…) to draw and transform shapes

Measure

The key York St John messages and associated research findings

'One way of becoming a better estimator is to learn by heart the size of some specific **reference items**. Children should be encouraged to do this for length, mass and capacity, and then to relate other estimates to these.... [e.g.] The mass of an individual packet of crisps is 30g' (Haylock, 2010, p288)

'Children should develop their skills and understanding of measurement through practical, purposeful activities. They should learn to choose and use appropriate measuring devices, discussing the ideas of accuracy and approximation.' (Haylock, 2010, p. 278)

'...children will need to go through the experience of using non-uniform, non-standard units such as pencils, crayons and body measurements like footprints...By using non-standard units to compare different measures, the children soon realise that the results can be unreliable. Through practical experiences children will discover the need for uniform non-standard units such as multilink. '(Dudgeon, 2005, p105)

'Teachers need to consider carefully... how to use possible misconceptions that arise from [measurement] opportunities to develop their pupils.' (Dudgeon, 2005, p104)

- **Take time to develop children's estimate skills, remember this is not the same as 'measuring'. The children need to understand that to make a sensible estimation they have to use some kind of a known/familiar 'benchmark'.** ***(e.g. knowing the mass of a bag is sugar is 1 kg and using this to compare another object against)***

- **When teaching conversions between different types of metric units highlight the links with place value and the use of suffixes as clues.** ***(e.g. 'centi' indicates one hundredth) – try to provide a visual or physical model e.g. constructing a metre cubed using balloons)***
- **The teaching of measurement should always be done within a practical context.**
- **Don't take for granted children's skills in using measuring equipment – this needs careful modelling; use of particular types of scales and intervals should be matched against their level of work in number.**
- **Enable children to use a variety of appropriate scales, linked to measuring equipment. Don't just use whatever the school has without checking if it is appropriate.**
- **Consider not just the learning but also their skills in using equipment.**
- **Ensure a sensible progression in learning measures. Don't begin with standard measures unthinkingly. After some structured play to introduce the concept and basic vocabulary move on to direct comparison before asking them to measure using non-standard, then standard units.**
- **Introduce examples to challenge misconceptions.** ***(e.g. provide small heavy items when covering mass; tall thin and short fat containers that hold the same amount when covering capacity)***
- **Remember to cover a range of measurements over time – not solely focusing on length.**
- **Do not cover more than one concept of measurement at any one time as it will confuse children.**

Here are some prompts from the DCSF publications ' Children thinking mathematically: PSRN essential knowledge for Early Years Practice' and 'Securing Levels 2 & 4 in Mathematics' to support in providing subject specific feedback and targets for the teaching of measurement.		
'Children thinking mathematically: PSRN essential knowledge for Early Years Practice' **Make sure that:** • in Early Years settings you use daily routines, ordering, predicting what comes next and recalling what they have done earlier to help young children to become increasingly aware of time • you talk to young children about activities in natural contexts such as the time left before snack time; growing seeds; discussing how long a cake needs in the oven; waiting for eggs to hatch; discussing how many bedtimes before a child's birthday to help link concepts of time and its language • you encourage play that involves movement, such as walking and running; dance, movement and music; picture story books relating to time and age to provide a broad experience in this area • you provide children with a range of interesting measuring materials and different lengths of ribbon or string, including real measuring devices to explore • you observe the children and find out what they are interested in; for example children may be interested in filling containers (investigating ideas of capacity and volume) • you consider that children may be interested in measuring heights. Height charts should include standard and non-standard measures, such as handprints, and give children opportunities to write their own numbers	***'Securing level 2 in mathematics'*** **Make sure that:** • measurement activities are included in oral and mental work • you provide opportunities to discuss the units used to measure length, mass, capacity and time • children have opportunities to discuss and compare measures, using uniform non-standard units, enabling them to recognise the need for having standard units of measure • children have opportunities to make their own scales, using uniform measures • children understand how they can use their number line skills to help them read scales • you draw on cross-curricular opportunities to offer children practical opportunities to apply their measurement skills in context • children hear and use the language of measures when measuring, estimating, comparing and sorting	***'Securing level 4 in mathematics'*** **Make sure that:** • you regularly use scales as part of your daily oral and mental work • children have regular opportunities to solve problems that involve reading scales; encourage them to explain their methods orally and in writing • scales are displayed and accessible so that children can handle and use them regularly • children are taught to identify where a scale starts and ends and how to use division to find what the interval size represents • children have a strategy to check estimated values by counting up and down the intervals • children understand the link between number lines and scales; count up and down scales, including using alternative units, for example: 0g, 100g, 200g…; 0kg, 0.1 kg, 0.2 kg… • children are expected to annotate scales and write in missing values on unnumbered marks • children use scales in different orientations; show them how scales can be rotated if this is helpful

Handling Data

The key York St John messages and associated research findings

'Learning to sort data according to given criteria is the foundation of counting and also of data handling. Give younger children lots of experience of sorting, first using actual objects themselves.' (Haylock, 2010, p342)

'A teacher committed to promoting meaningful learning and to motivating pupils through purposeful activities will...plan for children to be involved in all the stages of data handling – not just drawing a graph, but involved in determining what data should be collected, actually collecting it and recording it, organising it, representing it and finally, interpreting it.' (Haylock & Cockburn, 2003, p185)

'The problem with most data-handling experienced by pupils in primary schools is that it is dreadfully dull. Typically, the pupils are presented with a list of numbers in a textbook or on a worksheet and are instructed to arrange the numbers in order... The publisher of the book or the teacher who created the worksheet pays lip-service to the notion of making the data-handling real by appending each number with 'cm' or 'kg'... This is not real data-handling. The pupils have no part in collecting the data, they have no ownership of the activity and almost certainly cannot see the point of all this sorting, searching, graphing and averaging.' (English, 2006, p63)

'Three ways to involve children in interpreting graphs: a) write about what the graph tells us ... b) write sentences about what the graph tells us, incorporating key words, such as most least, more than, less than etc. c) [children] make up a number of questions that can be answered from the graph, to pose to each other.' (Haylock, 2010, p345)

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- **Use ICT to cut down the leg-work on collecting and representing data. Ensure ICT enhances the mathematical learning.**
- **Ensure progression –avoid rushing on too soon to the more complex types of visual representations.**
- **Reduce focus on simply collecting data and ensure data is being collected for a good reason. Remember "motivation is higher when the data is collected by the children themselves, higher still when it is collected to answer some questions they have posed themselves and even higher when it is about themselves!" (Haylock, 2010, p346)**
- **Motivate children to plan their own investigation techniques rather than presenting them with what (and how) they are going to investigate.**
- **Start with a question the children are genuinely interested in.**
- **Make effective links with other areas of the curriculum and real life problem solving.**
- **Use a range of different representations of the data.**

Here are some prompts from the DCSF publications ' Children thinking mathematically: PSRN essential knowledge for Early Years Practice' and 'Securing Levels 2 & 4 in Mathematics' to support in providing subject specific feedback and targets for data handling.		
'Children thinking mathematically: PSRN essential knowledge for Early Years Practice' **Make sure that:** • children represent their own mathematical thinking in contexts that are relevant and meaningful to them – mathematical graphics • mathematical graphics begin in imaginative play, as they explore, make and communicate their personal meaning • you tune into and value children's own meaning • you value children's mathematical graphics • you discuss the children's mathematical graphics with them to help them find reflect on the meaning of their own and other children's graphics • observe closely and annotate children's mathematical graphics; this will allow adults to uncover children's thinking and meanings sensitively • model mathematical graphs indirectly – it is important for children to see different ways to represent mathematical thinking • create a culture in which children use graphical media independently throughout the environment and make a wide variety of paper, pens, pencils and other mark-making equipment, easily accessible in all provision, indoors and out	***'Securing level 2 in mathematics'*** **Make sure that:** • children have experience of sorting objects physically, for example, into hoops, before being asked to sort them onto diagrams such as Venn or Carroll diagrams, and choose their own criteria to classify objects • you show children how Venn and Carroll diagrams involving one criterion can be adapted to incorporate a second criterion • children have practical experience of creating pictograms and block graphs, through using counters or physical blocks, before moving into drawing graphs • you help children to understand the need for non-unit scales on graphs, for example, scales marked in 2s, 5s and 10s • children solve problems involving comparing or combining more than one piece of data, for example, how many more boys than girls are there in the class, how many children altogether? • children are given opportunities to work through the whole data-handling cycle, including suggesting a line of enquiry and discussing what data to collect and how • you draw on opportunities from other curricular areas and real-life opportunities to give children meaningful experiences of handling data	***'Securing level 4 in mathematics'*** **Make sure that:** • children annotate graphs, e.g. writing intermediate values along the axes • children have opportunities to interpret a wide range of tables/graphs/charts including: timetables, Carroll and Venn diagrams, pictograms, bar charts, bar line graphs, line graphs and pie charts • children are used to graphs shown in different orientations for example horizontal bar charts • children understand the importance of reading scales on graphs accurately and have the skills to do so • children have regular opportunities to answer questions that involve calculating with data, such as: How many more…? How many…altogether? • children have opportunities to make up their own questions based on a graph • you exploit opportunities to consolidate data handling skills through other subjects such as science

Success criteria in mathematics lesson plans:

A key message in mathematics (i.e. what we tell the student teachers and what we hope will be evidenced in their lesson planning) .....

Success criteria should not simply reiterate the learning objectives. They should provide you and the children with a list of explicit knowledge and skills that need to be used to provide evidence for the successful achievement of the objective(s). They may well be differentiated. To children they should be presented as the 'Top Tips' for successful completion of an activity and can be used at the start and end of the session to make the process clear and *should* be used at the end of the session to ensure that the key learning points have been reviewed.

NB There may be times when they are not shared at the start of the session if you do not want to 'spoil the surprise' about what you hope will be learnt. e.g. that prime numbers only have two factors OR strategies that can be used to solve a puzzle requiring the children to come up with a general rule for a sequence of numbers from a function machine.

In the table below are a number of examples of learning objectives that **may** be addressed in a mathematics lesson in a variety of different year groups. It is hoped that this exemplification is helpful about what is meant in our key message.

R LO - Use language such as greater, smaller, heavier, lighter to compare quantities
• Directly compare length by matching items exactly at one end (ALL) • Know that the bigger one protrudes (ALL) • Correctly attribute bigger/smaller/ longer/shorter (LAs) - also higher / taller (MAs) - also wider / narrower (HAs)
R LO – To be able to count 10 objects reliably
• Use 1:1 correspondence (i.e. point to each object once, matching each number name with an object, moving the object to one side when counted) (ALL) • Know correct stable order 1-10 (LAPS – 1-5) • Know that the last number stated is the number in the set (ALL) • Know that objects in a set do not always have to look identical (HAPs) • Know that the objects do not have to be counted in a set order (HAPs)
Y1 LO - Read the time to the hour and half hour
• Know that the long hand moves more quickly than the short hand and what each hand denotes (All) • Know that when the long hand points to the 12 it is 'o'clock' (All) - and when it points to the 6 it is 'half past' (MAs and HAs) • Know that at 'half past' the short hand will have moved half way to the next hour (HAs)
Y1 LO – Visualise and use everyday language to describe the position of objects and direction / distance when moving them
• Place item in correct position relative to a set point in response to an instruction • State correctly where the item is in relation to the point using over /under / behind /in front (LAs) - also above / underneath / at the side of (MAs) - also below and to the right / left of (HAs) OR • Give correct instructions for following an L shaped path using 'forward', 'left', 'right' (All) quarter turn (MAs and HAs) • Give a reasonable estimate for the number of 'robot' units between two points along a straight line (MAs & HAs) • Give correct instructions for re-tracing the L shaped path using 'backward','left', 'right', quarter turn (MAs) and 'forward' 'quarter turn' (HAs)
Y1 LO – Answer a question by recording information in lists and tables
• Decide on possible answers to a question (ALL) • Collect data using a tallying system (ALL) • Count up amounts (ALL) using multiples of 5 (MAPs and HAPs) • Correctly compare frequencies to say which is the most/least by identifying highest and lowest numbers (ALL) – how many more or less by counting on to find the difference (HAPs)

Y2 LO - Identify reflective symmetry in patterns and 2 D shapes

For 2D shapes:

- Know that a shape has reflective symmetry when it can be folded down a central line so that both sides match exactly on top of each other
- Recognise that the reflection of half of a shape in a mirror matches exactly the missing half of the shape (or that the given half, plus the reflected half, together make the original shape)
- Know that the half of the shape seen in the mirror has been 'flipped' and is the reverse of the part seen (HAPs)

For patterns....

- Know that each part of the pattern on one half is the same distance from a central mirror line as the equivalent part in the reflection
- Position the reflection of a part of the pattern by counting squares

Y2 LO - Use informal methods to add (or subtract) 2 digit numbers

For addition:

- Partition into tens and ones
- Add both sets of tens and then both sets of ones separately
- Recombine the two parts

or

- Mark the higher of the numbers on an empty number line
- Partition the smaller number into tens and ones
- Count on the tens from the higher number then count on the ones (HAs should be able to use number bonds of ten to count on the tens in one step)

For subtraction:

- Recognise which is the lower of the two numbers
- Count on to find the difference from the lower to the higher number using an empty number line in as few jumps as possible (i.e. via marking the lower number; counting on in tens to the multiple below the target and then in ones to the target; then combining the two jumps)

Y4 LO - Calculate time intervals from clocks

- Read times to the nearest minute on an analogue clock
- Using an empty number line and clock face - Count on from time A in hours to the time nearest to, and before, time B
- Count on in minutes to the nearest 5 minute interval then in fives to time B
- Combine these amounts to give the total time interval
- LAs - as above but using a clock face only

Y4 LO - Choose and use standard metric units and abbreviations when estimating, measuring and recording length

- Make a sensible estimate of the length of straight lines (<30 cms) (All) - also curved lines (HAs)
- Know that cm = centimetre (All)
- Use a 30cm ruler to accurately measure straight lines to the nearest centimetre, positioning the '0' mark at the very start of the line (ALL)

Y4 LO – Answer a question by interpreting the data in a bar chart

- Know that the most / least common is the tallest / shortest bar (ALL)
- Utilise the Y axis scale to identify how many items are represented by any one bar (MAPS and HAPs)
- Use counting on to find the difference in order to find out how many more/less represented by two bars (HAPS)

Y4 LO - Interpret mixed numbers and position them on a number line'

- Know that mixed numbers must first be turned into improper fractions before ordering
- Recognise which part of the number is an integer and which part is a fraction
- Multiply the integer by the fraction's denominator then add the additional fraction when doing this transformation
- Know equivalent fractions within the ½, ¼, 1/8 and the 1/10 and 1/100 families and use these to transform the fractions further
- Transform the improper fractions back to their original mixed number format after ordering, using the inverse operation

Y5 LO - To understand percentages as the number of parts in every 100 and express tenths and hundredths as percentages

- To know that 'percentage' means 'out of 100' and relate this to the proper fraction equivalent (e.g. move from 30% to 30/100)
- To be able to convert the proper fraction into its decimal equivalent by dividing the numerator by the denominator
- To know that 'x100' can be read as 'of 100' (this is crucial in helping them understand how to turn a fraction into a % e.g. ¾ as a % means ¾ of 100 or ¾ x 100)
- To know that to change a decimal (or proper) fraction into its % equivalent you need to x 100
- To be able to multiply a decimal fraction by 100 by moving the digits to two places to the left
- (For more able children) To be able to convert any given proportion (e.g. 35 people in a group of 50) into a % by expressing as a proper fraction and then multiplying by 100